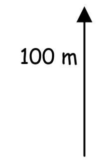
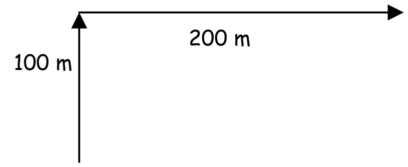
Vector Introduction

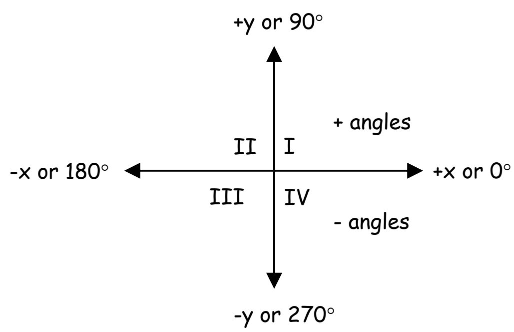
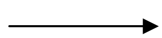
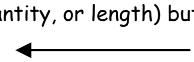
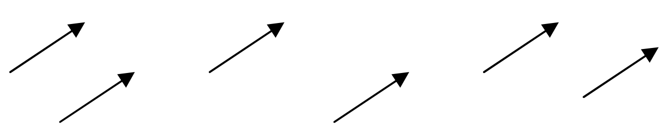
Remember this section is an introduction for 70 to 80 percent of students entering this class. Even those who say they have been taught vectors rarely understand them well enough. All students should attempt this section to the best of their abilities.

The easiest vector to understand is the displacement vector, measured in meters. If I say, “go 100 meters north” most people can visualize what that event would look like. A vector is a representation of that event. I would draw an arrow pointing north, like the one at the right. The length of the arrow is up to me, but once I draw this vector any other vectors must match this scale. The length you see at the right now represents 100 meters.

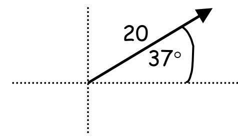
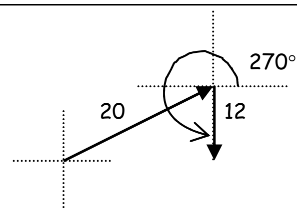
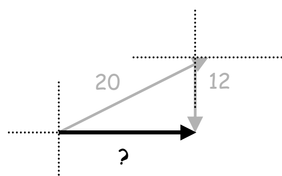


If you went 100 meters north and were then told to go 200 meters east this arrow would point east. It would be drawn twice as long as the first vector since 200 meters is twice as big as 100 meters. These two vectors are shown at the right. In essence giving directions to get to your house is very much like a vector problem. You tell someone to travel a certain direction (along a street) for a certain distance (2 miles), etc. Drawing vectors is like drawing a map to your house. Except that vectors only come in straight lines.

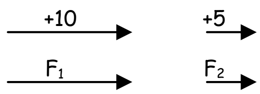
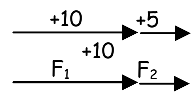
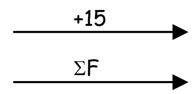
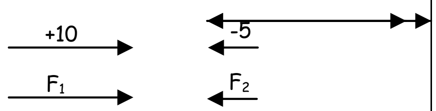
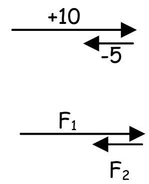
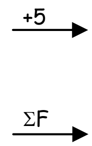
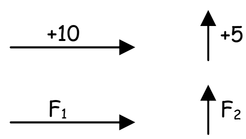
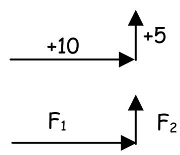
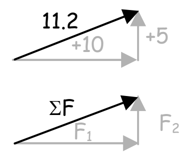
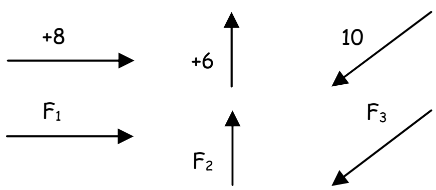
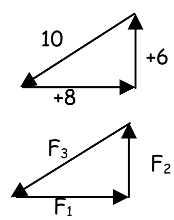
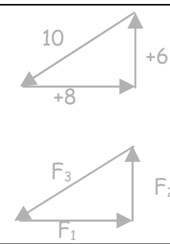


Vector:	a physical quantity with ***both*** a ***magnitude*** and a ***direction***.
Magnitude:	Size or extent. The numerical value. In the diagrams 100 m and 200 m are the magnitudes of the vectors.
Direction:	Alignment or orientation. North and East are the directions.
Scalars:	A quantity described by ***magnitude only***. If there is no direction, it is a scalar.
Notation:	Vectors can be represented in several ways. The following are all ways to represent a vector **X** which is equal to 50 meters directed east. The two on the left would be used in text describing the vector, while the two on the right are diagrams. **X** $\vec{X}$  The AP exam and equation sheets used throughout the year have vector variables in bold type. A letter shown in italics represents scalars or just the magnitude of a vector. The second symbol may appear from time to time in the text or in other Physics courses. The figure at the right shows the arrow representing the vector and substitutes the numerical equivalent for *X*.
Refining Direction:	North, south, east, and west are acceptable directions for vectors, and they work great when giving someone directions for moving along the Earth’s surface. In Physics there is a more accepted directional system. ***We will use the mathematical coordinate axis system.***  Refer to the coordinate axis shown at the right. If an object is moving to the right we can say that it is moving in the +*x* direction or we can also say that it is moving at 0o. Typically if the vector lies along an axis we report +*x*, -*x*, +*y*, or +*y*. Use degrees for vectors that are at an angle to the coordinate axis system. Positive angles are measured from +*x* (0°) in the counterclockwise direction. This follows the numbering convention of the quadrants which increase in the counterclockwise direction. A negative angle would be measured clockwise.
Negative Vectors:	A vector with the ***same magnitude*** (numerical quantity, or length) but ***opposite direction***. If vector **A** looks like this  then vector -**A** looks like this 
Equal Vectors:	All of the following are equal vectors. It does not matter that they start and end at different locations. Equal vectors must have the same magnitude (numerical value or length) and must have the same direction (point the same way). 

Basic Vector Addition:	You can add two or more vectors. The result of adding vectors is known as the vector ***resultant*** or the ***sum of vectors***. If you are adding vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, the resultant would be the "sum of **F**" written as $\Sigma\mathbf{F}$. The symbol Σ is the Greek letter sigma used as an abbreviation of the words "sum of". The preferred method of vector addition is tip to tail. Sometimes you are given an arrow and a variable label and other times you will be given magnitude and direction. Either way the tip to tail method is as follows :

1	Imagine a coordinate axis where you want to draw the vector. Start the tail of the vector at the origin of the axis. Draw the tip of the vector pointing in the direction specified. You pick the length, but now you have set the scale. Any other vectors drawn should be roughly proportional to this one.	***Example:*** Find the magnitude and direction when $\mathbf{F}_1$ = 20 m at 37° to $\mathbf{F}_2$ = 12 m at 270° are added together. 
2	The next vector begins at the tip of the first vector. So imagine a coordinate axis appearing at the tip of the arrow. Start the tail of the second vector at this new coordinate axis. Since this vector's tail begins at the tip of the first vector the method is called tip to tail. Repeat this process for all remaining vectors.	
3	The resultant **is not** drawn tip to tail (from the tip of the last vector to the tail of the first.). Instead it is drawn from the start of the problem to the end of the problem. Think of it as the shortest possible path from the beginning to the end.	
4	Use sine, cosine, tangent, Pythagorean Theorem, etc. to find the magnitude (length) and direction (angle) of the resultant.	Since there is a 37° angle I know that this is a 3,4,5 triangle. I can deduce the length of the final side as 16. I also know that it points down the $+x$ axis (0°). The magnitude and direction are16 m $+x$ or 16 m at 0°

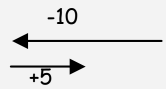
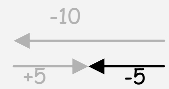
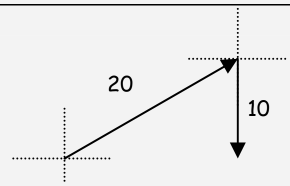
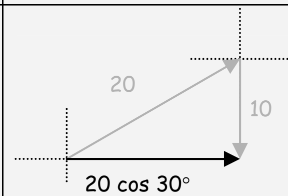
The following are common examples of vector addition shown in number and variable form.

	Vectors to be Added	Adding them tip to tail	Resultant (sum)
Two vectors pointing the same way			 $\Sigma F = F_1 + F_2$
Two vectors pointing in opposite direction			 $\Sigma F = F_1 - F_2$
Two perpendicular vectors are added			 $\sqrt{10^2 + 5^2} = 11.2$ $\Sigma F = \sqrt{F_1^2 + F_2^2}$
Three vectors are added			 The sum is 0 $\Sigma F = 0$

Finding magnitude (length) is either simple addition or Pythagorean Theorem. Often, the vectors you are looking for are on an axis and you can use *+x*, *-x*, *+y*, and *-y* for direction. Otherwise you can use sine, cosine, and tangent to find the angles (directions). Sometimes, vectors added tip to tail return to the origin and the vector sum is zero. Occasionally you are given the resultant and asked to find one of the vectors that created it.

You will see later in the course that all vector problems, even the most difficult ones, can actually be turned into one of the four easy examples shown above.

Problem Set: Solve the following vector problems. Remember, you can only use +*x*, -*x*, +*y*, and -*y* when a vector is on that axis. If a vector is listed as 10 in the +*x* direction, then it is mathematically treated as +10. If a vector is listed as 10 in the –*x* direction, then it is mathematically treated as -10. Vectors not on the axis must have their direction given in degrees, such as 10 at 30°. Use sine, cosine, tangent and Pythagorean Theorem to find angles (direction) and / or lengths (magnitude).

Problem	Tip to Tail	Resultant (sum of vectors)	Formal Answer
Example Add 10 in the -*x* 5 in the +*x*			5 in the -*x* direction
Example Add 20 at 30° 10 at 270°			17.3 at 0° or 17.3 in +*x* direction
a. Add 10 in the +*x* 10 in the-*x* 5 in the +*y*			
b. Add 5 in the +y 10 in the –x 5 in the –y			
c. Add 40 in the +y 30 in the –x			
d. Add 6 in the +y 8 in the +x and a third vector add to zero. What is the third vector?			
e. Add 10 at 60° 5 at 180°			
f. Add 15 at 37° 9 at 270° 12 at 180°			

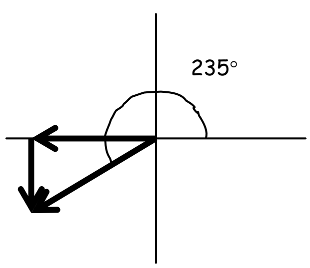
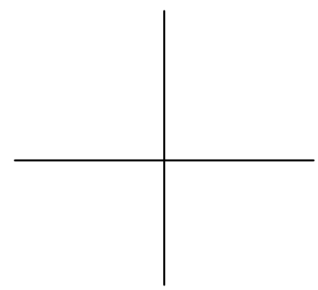
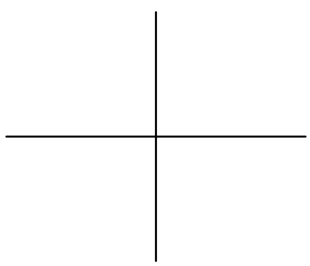
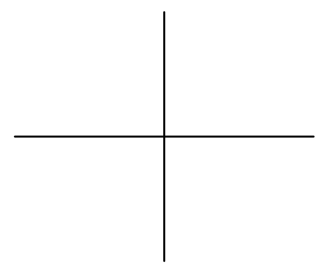
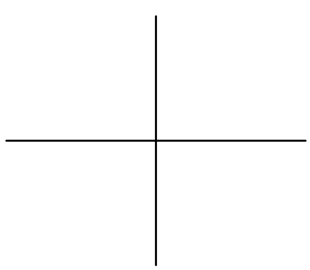
Vector Components

A resultant vector is a vector resulting from the sum of two or more other vectors. Mathematically, the resultant has the same magnitude and direction as the total of the vectors that compose the resultant.

Component Vectors

Could we reverse this logic? Could we use two or more vectors to describe another vector? Yes we can.
There is probably more than one way to get to your house. In vector terms, all paths arriving at the same destination are equal. They accomplish the same thing. The resultant is just the shortest possible path from start to finish. So if we are given a vector we can use two vectors that follow a different path but arrive at the same location to take its place. Vector **R** in the diagram at the right can be replaced by vectors **R***x* and **R***y*. But, why would we bother? **R***x* and **R***y* lie on the coordinate axes. That is why they have *x* and *y* subscripts. We have seen earlier that it is very easy to add vectors on the coordinate axis (Example 10 in the +*x* and 5 in the +*x* equals 15 in the +*x*). Vectors on an axis require simple addition and subtraction for the magnitude and + and □□ for direction. **Vectors on an axis that add up to another vector are known as the component vectors**. Essentially **R***x* is the width of **R**, and **R***y* is the height of vector **R**. The sign on the component vectors depends on the quadrant: I (+*x* +*y*), II (-*x* +*y*), III (-*x* -*y*), and IV (+*x* -*y*). There is another advantage to component vectors: they complete a right triangle. Again, sine, cosine, and tangent and Pythagorean Theorem will come in handy.

Problem Set: Solve the following problems. If you are given a vector, draw it and its components. Then calculate the magnitude of the *x* and *y* components. If you are given the components, solve for the parent vector's magnitude and direction.

Example: 250 at 235°  $x = hyp\ \cos\theta$ $= 250\cos 235^\circ$ $= \boxed{-143}$ $y = hyp\ \sin\theta$ $= 250\sin 235^\circ$ $= \boxed{-205}$	b. 89 at 150° 
b. 6.5 at 345° 	c. 12 at 265° 
d. 7.5E4 at 180° 	e. 0.00556 at 60°